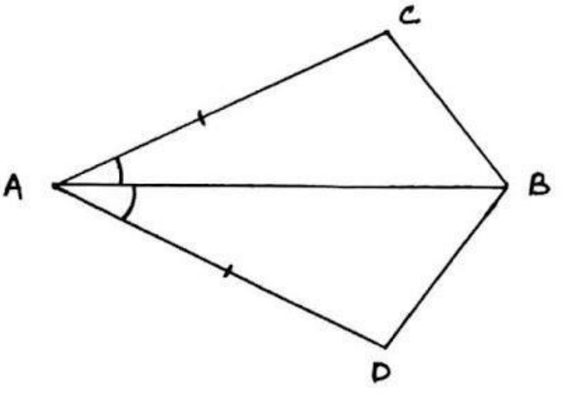
TRIANGLE

Q1 In quadrilateral ABCD $AC = AD$ and AB bisects $\angle A$. Show that $\triangle ABC \cong \triangle ABD$. What can you say about BC and BD?

Sol: → In $\triangle ABC$ and $\triangle ABD$, we have



$AC = AD$ [Given]

$\angle CAB = \angle DAB$

$AB = AB$

$\therefore \triangle ABC \cong \triangle ABD$

[By SAS congruence]

Therefore $BC = BD$ (CPCT) Ans.

Q2 ABCD is a quadrilateral in which $AD = BC$ and $\angle DAB = \angle CBA$

Prove that

i) $\triangle ABD = \triangle BAC$

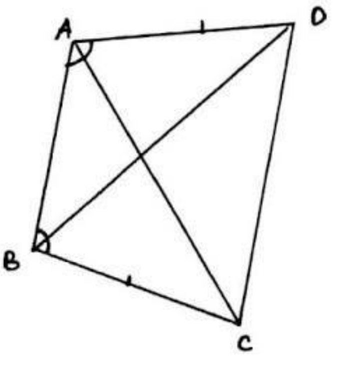
ii) $BD = AC$

iii) $\angle ABD = \angle BAC$

Sol: → In the given figure ABCD is a quadrilateral in which

$AD = BC$ and $\angle DAB = \angle CBA$

In $\triangle ABD$ and $\triangle BAC$, we have



$AD = BC$ [Given]

$\angle DAB = \angle CBA$ "

$AB = AB$ [Common]

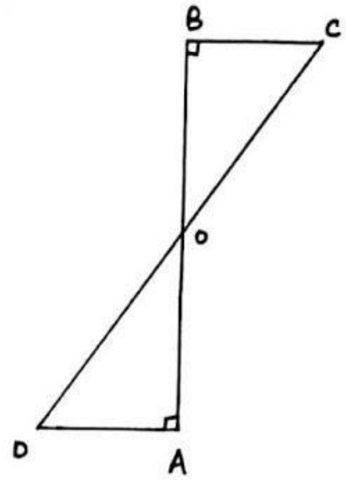
$\therefore \triangle ABD \cong \triangle BAC$ [By SAS]

$\therefore BD = AC$ [CPCT]

and $\angle ABD = \angle BAC$ [CPCT]

Proved

Q3 AD and BC are equal perpendiculars to a line segment AB. Show that CD bisects AB.



Sol:→ In ΔAOD and ΔBOC we have

$\angle AOD = \angle BOC$ [vertically oppsite angles]

$\angle CBO = \angle DAO$ [Each $= 90°$]

and $AD = BC$ [Given]

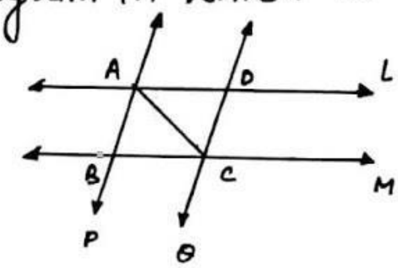
$\therefore \Delta AOD \cong \Delta BOC$ [By AAS Congruence]

Also $AO = BO$ [CPCT]

Hence, CD bisects AB

Proved.

Q4 l and m are two parallel lines intersected by another pair of parallel lines p and q Show that $\Delta ABC \cong \Delta CDA$



Sol. In the given figure, ABCD is a parallelogram in which AC is a diagonal ie AB||DC and BC || AD

In ΔABC and ΔCDA we have

$\angle BAC = \angle DCA$ [Alternate angle]

$\angle BCA = \angle DAC$,,

$AC = AC$ [common]

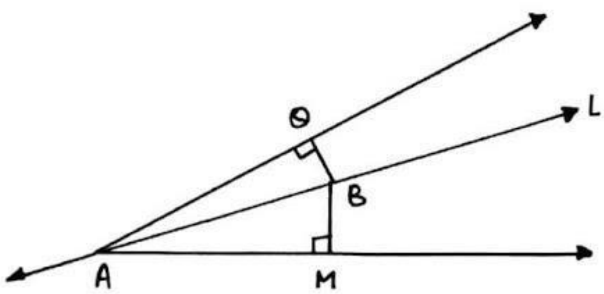
$\therefore \Delta ABC \cong \Delta CDA$ [By ASA congruence]

Proved

Q5 Line l is the bisector of an angle A and B is any point on l, BP and BQ are perpendicular from B to the arms of $\angle A$ (see fig) Show that

i) $\Delta APB \cong \Delta AQB$

ii) $BP = BQ$ or B is equidistant from the arms of $\angle A$



Sol:→ In ΔAPB and ΔAQB, we have

$\angle PAB = \angle QAB$ [l is the bisector of $\angle A$]

$\angle APB = \angle AQB$ [Each 90°]

$AB = AB$ [Common]

$\therefore \Delta APB \cong \Delta AQB$ [By AAS Congruence]

Also $BP = BQ$ By CPCT

ie B is equidistant from the arms of $\angle A$

Hence Proved

Q6 In the figure, $AC = AE$, $AB = AD$ and $\angle BAD = \angle EAC$. Show that $BC = DE$.

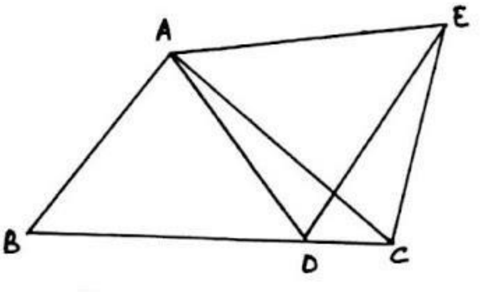
Sol:→ $\angle BAD = \angle EAC$ [Given]

$\Rightarrow \angle BAD + \angle DAC = \angle EAC + \angle DAC$

[Adding $\angle DAC$ to both sides]

$\Rightarrow \angle BAC = \angle EAC$ —(i)

Now in ΔABC and ΔADE, we have



$AB = AD$ [Given]

$AC = AE$ [Given]

$\Rightarrow \angle BAC = \angle DAE$ [From i]

$\therefore \Delta ABC \cong \Delta ADE$ [By SAS congruence]

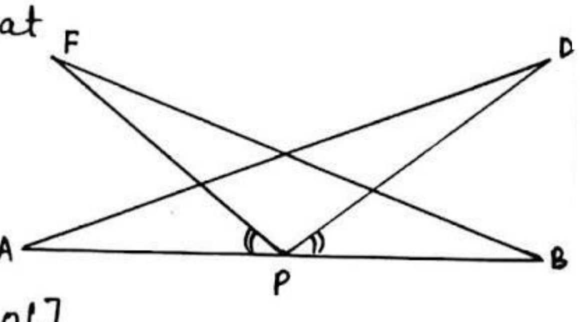
$\Rightarrow BC = DE$ [By CPCT]

Hence proved.

Q7 AB is a line segment and P is its mid-point D and E are points on the same side of AB such that $\angle BAD = \angle ABE$ and $\angle EPA = \angle DPB$ [See fig] Show that

i) $\Delta DAP \cong \Delta EBP$

ii) $AD = BE$



Sol :→ In ΔDAP and ΔEBP, we have

$AP = BP$ [$\because P$ is mid-point of]

[Line segment AB]

$\angle BAD = \angle ABE$ [Given]

$\angle EPB = \angle DPA$

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$\therefore \quad \Delta BPA \cong \Delta EPB$ [ASA]

$\Rightarrow AD = BE$ [By CPCT]

Hence proved

Q8 In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B

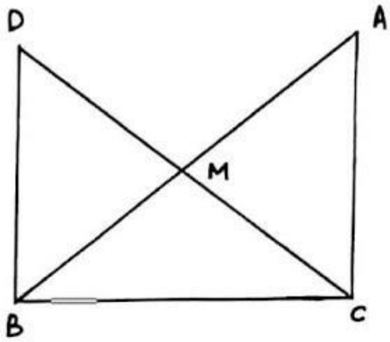
Show that:

i) $\Delta AMC \cong \Delta BMD$

ii) $\angle DBC$ is a right angle.

iii) $\Delta DBC \cong \Delta ACB$

iv) $CM = \frac{1}{2} AB$



Sol :→ In ΔBMD and ΔAMC

$DM = CM$ [Given]

$BM = AM$ [M is the mid-point of AB]

$\angle DMB = \angle AMC$ [V.O.A]

$\therefore \quad \Delta AMC \cong \Delta BMD$ [By SAS]

Hence Proved.

ii) $AC \parallel BD$ [$\angle DBM$ and $\angle CAM$ are alternate angle]

$\Rightarrow \angle DBC + \angle ACB = 180^\circ$ [sum of co-interior angles]

$\angle DBC + \angle ACB = 180$ [$\angle ACB = 90^\circ$]

$\therefore \quad \angle DBC = 90^\circ$

Hence proved.

iii) In ΔDBC and ΔACB, we have

$DB = AC$ [CPCT]

$BC = BC$ [common]

$\angle DBC = \angle ACB$ [Each 90°]

$\therefore \quad \Delta DBC \cong \Delta ACB$ [By SAS] Proved

iv) As $\triangle DBC \cong \triangle ACB$

$\therefore AB = CD$ [By CPCT]

$\Rightarrow \frac{1}{2}AB = \frac{1}{2}CD$

Hence, $\frac{1}{2}AB = CM$ $[CM = \frac{1}{2}CD]$

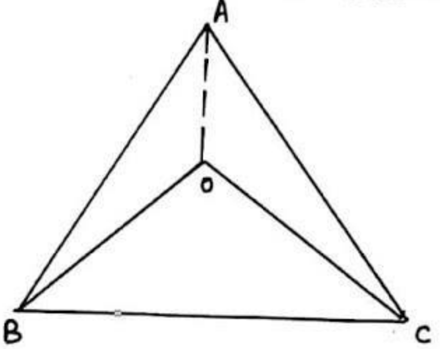
Hence proved.

Exercise 7.2

Q1 In an isosceles triangle ABC, with AB = AC, the bisector of $\angle B$ and $\angle C$ intersect each other at O. Join A to O Show that

i) OB = OC

ii) AO bisect $\angle A$



Sol. (i) AB = AC (Given)

$\Rightarrow \angle ABC = \angle ACB$

$\frac{1}{2}\angle ABC = \frac{1}{2}\angle ACB$

$\Rightarrow \angle CBO = \angle BCO$ [OB and OC are bisector of $\angle B$ and $\angle C$]

$\Rightarrow OB = OC$ [Sides opposite to equal angles are equal]

Again $\angle \frac{1}{2}ABC = \frac{1}{2}\angle ACB$

$\Rightarrow \angle ABO = \angle ACO$ [$\because$ OB and OC are bisector of $\angle B$ and $\angle C$ respectively]

In $\triangle ABO$ and $\triangle ACO$, we have

AB = AC [Given]

OB = OC [Proved above]

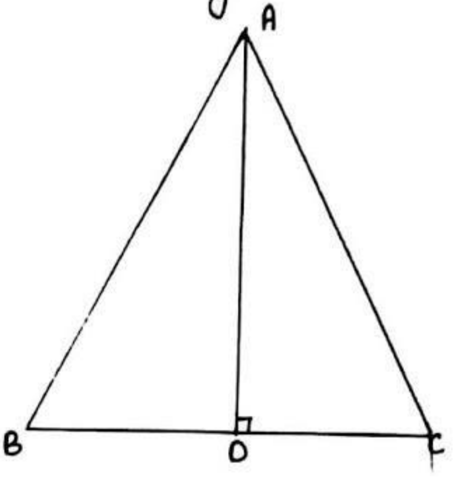
$\angle ABO = \angle ACO$ ["]

$\therefore \triangle ABO \cong \triangle ACO$ [SAS congruence]

$\Rightarrow \angle BAO = \angle CAO$

$\Rightarrow$ AO bisect $\angle A$ Hence proved.

Q2 In ΔABC, AD is the perpendicular bisector of BC. (see figure). Show that ΔABC is an isosceles triangle in which $AB = AC$.



Sol:→ In ΔABD and ΔACD, we have

$\angle ADB = \angle ADC$ [Each 90°]

$BD = CD$ [AD bisect BC]

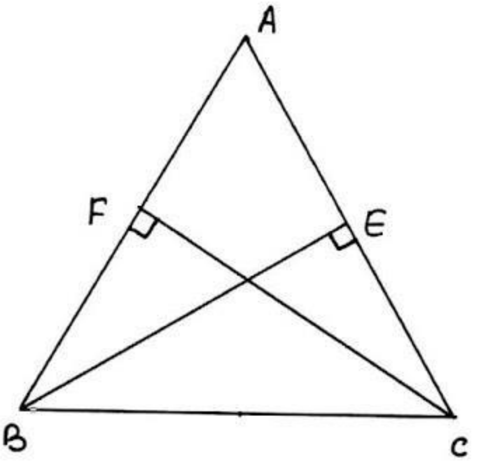
$AD = AD$ [Common]

$\therefore \Delta ABD \cong \Delta ACD$ [SAS]

$\therefore AB = AC$ [By CPCT]

Hence ΔABC is an isosceles triangle Proved.

Q3:→ ABC is an isosceles triangle in which altitude BE and CF are drawn to equal sides AC and AB respectively. Show that these altitudes are equal.



Sol:→ In ΔABC

$AB = AC$ [Given]

$\Rightarrow \angle B = \angle C$ [Angles opp to equal Sides of triangle are equal]

Now, in right triangles BFC and CEB

$\angle BFC = \angle CEB$ [Each 90°]

$\angle FBC = \angle ECB$ [Proved above]

$BC = BC$ [Common]

$\therefore \Delta BFC \cong \Delta CEB$ [AAS]

Hence, $BE = CF$ [CPCT]

Q4 ABC is a triangle in which altitude BE and CF to sides AC and AB are equal show that

i) $\Delta ABE \cong \Delta ACF$

ii) $AB = AC$

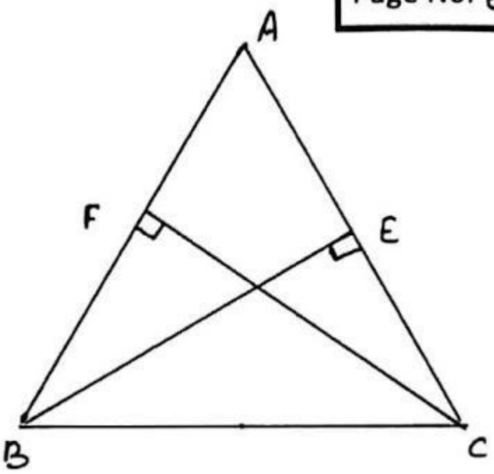
Sol :→ (i) In ΔABE and ACF we have

$BE = CF$ [Given]

$\angle BAE = \angle CAF$ [Common]

$\angle BEA = \angle CFA$ [Each 90°]

So $\Delta ABE \cong \angle ACF$ [AAS]



(ii) Also $AB = AC$ [By CPCT]

ie ABC is an isosceles triangle.

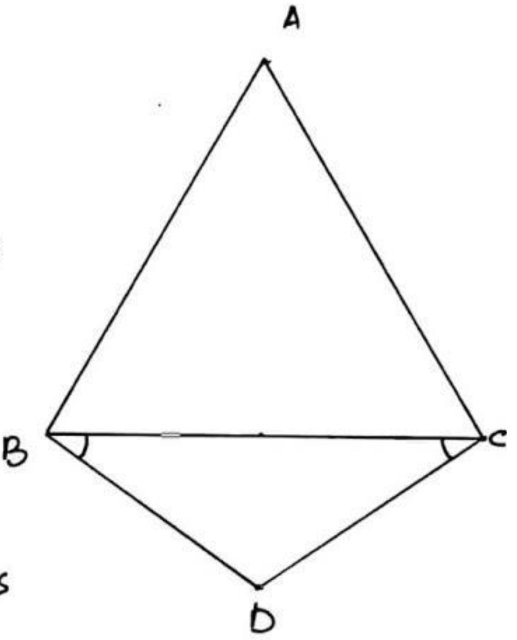
Q5 ABC and DBC are two triangles on the same base BC. Show that $\angle ABD = \angle ACD$

Sol :→ In isosceles ΔABC, we have

$AB = AC$

$\therefore \angle ABC = \angle ACB$ —(i)

[Angles opp to equal sides are equal]



Now, in isosceles ΔDCB, we have

$BD = CD$

$\angle DBC = \angle DCB$ —(ii)

[Angles opp to equal sides are equal]

Adding (i) and (ii) we have

$\angle ABC + \angle DBC = \angle ACB + \angle DCB$

$\Rightarrow \angle ABD = \angle ACD$

Proved.

Q6 :→ ΔABC is an isosceles triangle in which $AB = AC$ Side BA is produced to D such that $AD = AB$. Show that $\angle BCD$ is a right angle.

Sol :→ $AB = AC$ [Given]

$\angle ACB = \angle ABC$ — (i)

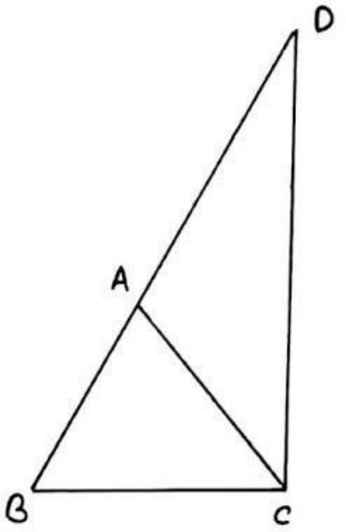
[Angles opp to equal sides are equal]

$AB = AD$ [Given]

$\therefore AD = AC$ [$\because AB = AC$]

$\therefore \angle ACD = \angle ADC$ —(ii)

[Angles opp. to equal sides are also equal]



Adding (i) and (ii)

$\angle ACB + \angle ACD = \angle ABC + \angle ADC$

$\Rightarrow \angle BCD = \angle ABC + \angle ADC$ — (iii)

Now in ΔBCD, we have

$\angle BCD + \angle DBC + \angle BDC = 180°$ [Angle sum property of a triangle]

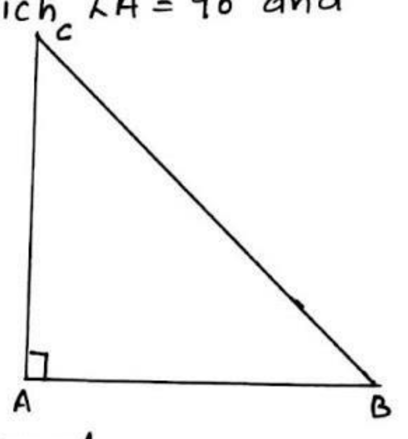
$\therefore \angle BCD + \angle BCD = 180°$

$2\angle BCD = 180°$

$\therefore \angle BCD = 90°$

Hence $\angle BCD = 90°$ or a right angle

Q7 ABC is a right angled triangle in which $\angle A = 90°$ and $AB = AC$ Find $\angle B$ and $\angle C$.



Sol :→ In ΔABC, we have

$\left.\begin{array}{r}\angle A = 90° \\ \text{and } AB = AC\end{array}\right\}$ [Given]

We know that angles opposite to equal sides of an isosceles triangle are equal.

So $\angle B = \angle C$

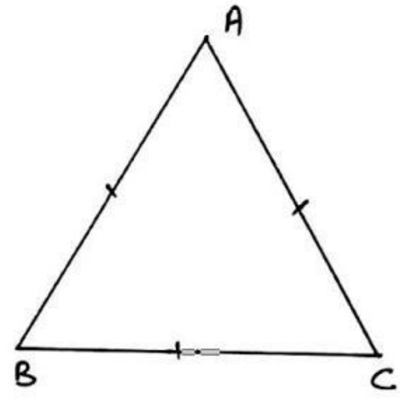
Since $\angle A = 90°$ therefore sum of remaining two angles $= 90°$

$\therefore \angle B = \angle C = 45°$

Q8 Show that the angles of an equilateral triangle are 60° each

Sol :→ As ΔABC is an equilateral triangle

So $AB = BC = AC$



Now $AB = AC$

$\therefore \angle ACB = \angle ABC$ —(i)

[Angles opp to equal sides are also equal]

Also $BC = AC$

$\Rightarrow \angle BAC = \angle ABC$ —(ii) [Same reason]

Now in ΔABC,

$\angle ABC + \angle ACB + \angle BAC = 180°$ [Angle sum property]

$\Rightarrow \angle ABC + \angle ABC + \angle ABC = 180°$ [From (i) & (ii)]

$\Rightarrow 3\angle ABC = 180°$

$\Rightarrow \angle ABC = \frac{\cancel{180°}}{\cancel{3}} = 60°$

Hence each angle of an equilateral triangle is 60°

Exercise 7.3

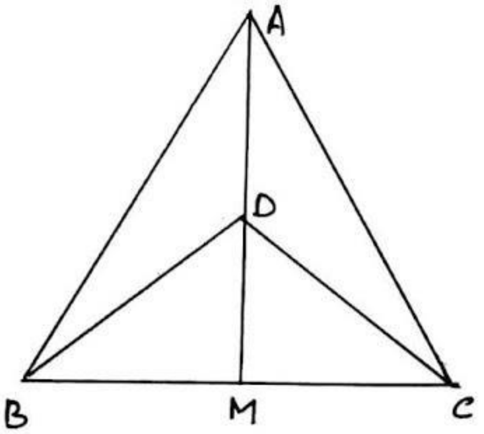
Q1 ΔABC and ΔDBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC. If AD is extended to intersect BC at P. Show that

i) $\Delta ABD \cong \Delta ACD$

ii) $\Delta ABP \cong \Delta ACP$

iii) AP bisect $\angle A$ as well as $\angle D$

iv) AP is the perpendicular bisector of BC



Sol In ΔABD and ΔACD, we have

$AB = AC$ [Given]

$BD = CD$ [Given]

$AD = AD$ [Common]

$\therefore \Delta ABD \cong \Delta ACD$ [SSS congruence] Proved —(i)

ii) In ΔABP and ΔACP, we have

$AB = AC$ [Given]

$\angle BAP = \angle CAP$ [$\because \angle BAD = \angle CAD$, by CPCT] —(ii)

$AP = AP$ [Common]

$\therefore \Delta ABP \cong \Delta ACP$ [SAS congruence]

iii) $\Delta ABD \cong \Delta ADC$ [From (i)]

$\Rightarrow \angle ADB = \angle ADC$ [CPCT]

$\Rightarrow 180^\circ - \angle ADB = 180^\circ - \angle ADC$

$\angle BDP = \angle CDP$

$\therefore$ $\boxed{\text{AP bisect } \angle D}$

Also from equation (ii)

$\angle BAP = \angle CAP$

$\therefore$ $\boxed{\text{AP bisect } \angle A}$

iv) Now $BP = CP$

and $\angle BPA = \angle CPA$ [By CPCT]

But $\angle BPA + \angle CPA = 180^\circ$ [Linear pair]

So $2\angle BPA = 180^\circ$

or $\angle BPA = 90^\circ$

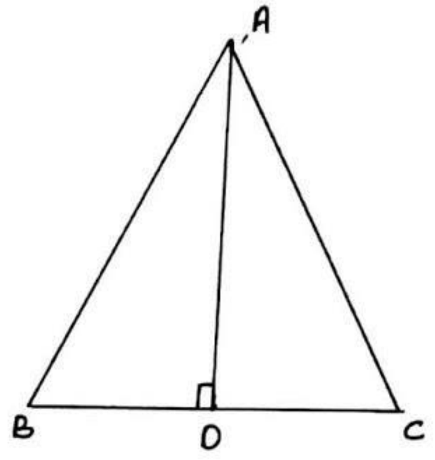
Since $BP = CP$, therefore AP is perpendicular bisector of BC.

Hence Proved.

Q2 AD is an altitude of an isosceles triangle ABC in which AB = AC Show that

i) AD bisects BC

ii) AD bisects $\angle A$



Sol:→ i) In $\triangle ABD$ and $\triangle ACD$, we have

$\angle ADB = \angle ADC$ [Each 90°]

$AB = AC$ [Given]

$AD = AD$ [common]

$\therefore \triangle ABD \cong \triangle ACD$ [RHS Congruence]

$\therefore BD = CD$ [CPCT]

Hence AD bisects BC

ii) Also $\angle BAD = \angle CAD$

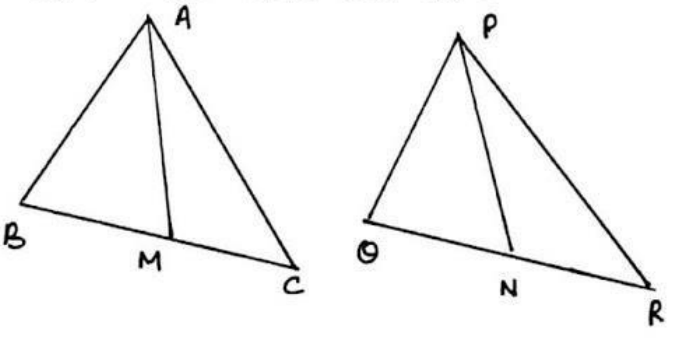
Hence AD bisects $\angle A$

Hence proved

Q3 :→ The sides AB and BC and median AM of one triangle ABC are respectively equal to sides PQ and QR and median PN of $\triangle PQR$ Show that

i) $\triangle ABM \cong \triangle PQN$

ii) $\triangle ABC \cong \triangle PQR$



Sol :→ In $\triangle ABM$ and $\triangle PQN$ we have

$BM = QN$ [$\because BC = QR$]

$\Rightarrow \frac{1}{2}BC = \frac{1}{2}QR$

$AB = PQ$ [Given]

$AM = PN$ [Given]

$\therefore \triangle ABM \cong \triangle PQN$ [SSS congruence]

$\Rightarrow \angle ABM = \angle PQN$ [CPCT]

Proved

ii) Now, in ΔABC and ΔPQR, we have

$AB = PQ$ [Given]

$\angle ABC = \angle PQR$ [Proved above]

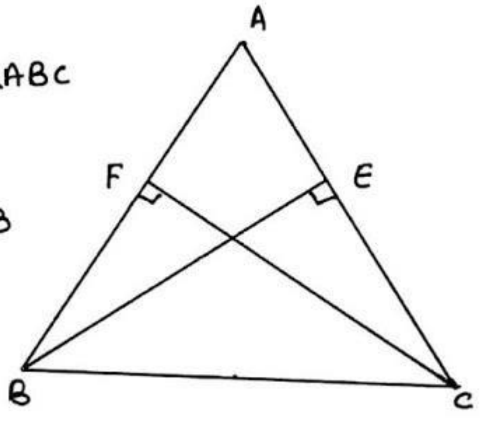
$BC = QR$ [Given]

$\therefore \Delta ABC \cong \Delta PQR$ [SAS congruence]

Proved

Q4:→ BE and CF are two equal altitudes of a triangle ABC Using R.H.S Congruence rule, prove that the triangle ABC is isosceles.

Sol:→ BE and CF are altitudes of a ΔABC



$\therefore \angle BEC = \angle CFB = 90^\circ$

Now, in right ΔBEB and CFB

$BC = BC$ [Common]

$BE = CF$ [Given]

$\therefore \Delta BEC \cong \Delta CFB$ [By R.H.S congruence rule]

$\therefore \angle BCE = \angle CBF$ [CPCT]

Now, in ΔABC, $\angle C = \angle B$

$\therefore AB = AC$ [side opposite to equal angles are equal]

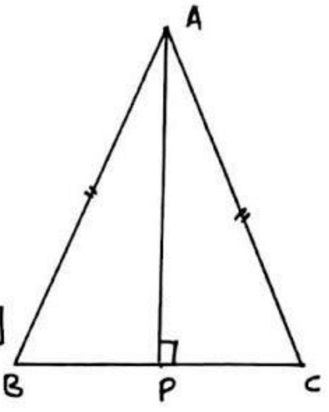
Hence, ΔABC is an isosceles triangle

Hence proved.

Q5:→ ABC is an isosceles triangle with $AB = AC$.
Draw $AP \perp BC$ to show that
$\angle B = \angle C$

Sol:→ Draw $AP \perp BC$

In ΔABP and ΔACP, we have



$AB = AC$ [Given]

$\angle APB = \angle APC$ [Each 90°]

$AB = AP$ [common]

$\therefore$ $\Delta ABP \cong \Delta ACP$ [By RHS Congruence rule]

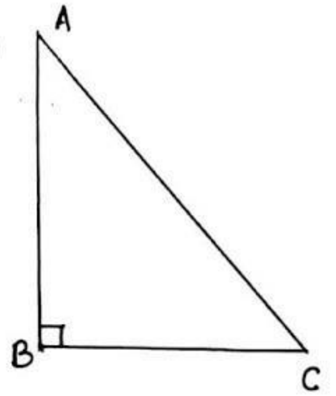
Also $\angle B = \angle C$

Hence Proved [CPCT]

Exercise 7.4

Q1. Show that in a right angled triangle, the hypotenuse is the longest side.

Sol:→ ABC is a right triangle, right angled at B



Now $\angle A + \angle C = 90^\circ$

$\Rightarrow$ Angles A and C are each less than 90°

Now $\angle B > \angle A$

$\Rightarrow$ $AC > BC$ —(i)

[Side opposite to greater angle is longer]

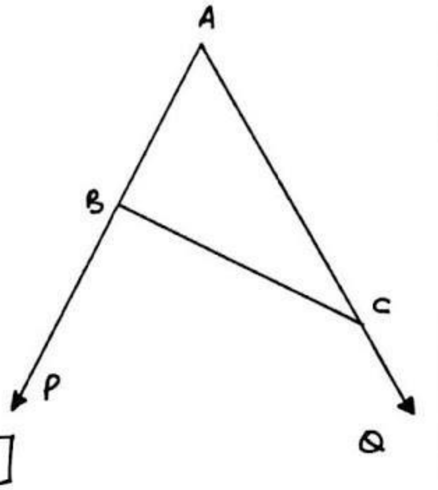
Again $\angle B > \angle C$

$AC > AB$ —(ii)

[Side opp to greater angle is longer]

Hence from (i) and (ii) we can say that AC (Hypotenuse) is the longest side

Q2 :→ In the figure Side AB and AC of ΔABC are extended to points P and Q respectively Also $\angle PBC < \angle QCB$. Show that $AC > AB$



Sol :→ $\angle ABC + \angle PBC = 180^\circ$ [Linear Pair]

$\Rightarrow \angle ABC = 180 - \angle PBC$ —(i)

Similarly $\angle ACB = 180^\circ - \angle QCB$ —(ii)

It is given that $\angle PBC < \angle QCB$

$\therefore 180^\circ - \angle QCB < 180^\circ - \angle PBC$

$\Rightarrow \angle ACB < \angle ABC$ [From (i) and (ii)]

$\Rightarrow AB < AC$

or $\boxed{AC > AB}$

Hence proved.

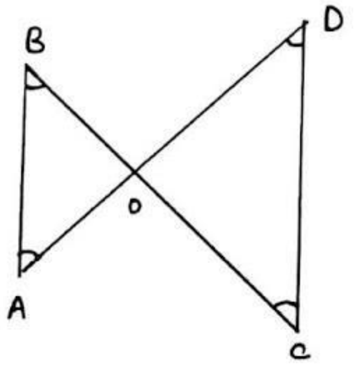
Q3 :→ In the figure $\angle B < \angle A$ and $\angle C < \angle D$. Show that $AD < BC$

Sol :→

$\angle B < \angle A$ [Given]

$\therefore BO > AO$ —(i)

[Side opp to greater angle is longer]



Also $\angle C < \angle D$

$\therefore CO > DO$ —(ii)

[Same reason]

Adding (i) + (ii) we get

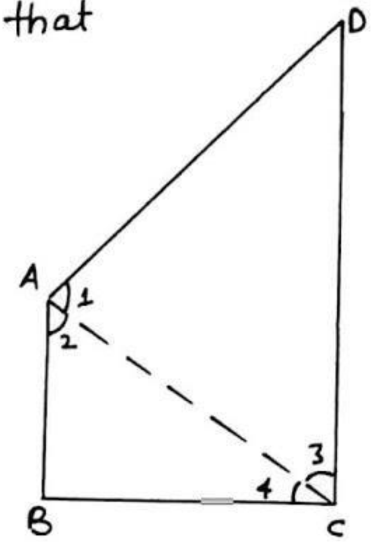
$BO + CO > AO + DO$

$BC > AD$

$AD < BC$

Hence Proved.

Q4 AB and CD are respectively the smallest and longest side of a quadrilateral ABCD. Show that $\angle A > \angle C$ and $\angle B > \angle D$



Sol :→ Join AC

Mark the angles as shown in fig.

In ΔABC

$BC > AB$ [AB is the shortest side]

$\Rightarrow \angle 2 > \angle 4$ — (i)

[Angle opp to longer side is greater]

In ΔADC

$CD > AD$ [CD is the longest side]

$\angle 1 > \angle 3$ — (ii)

[Angle opposite to longer side is greater]

Adding (i) and (ii) we have

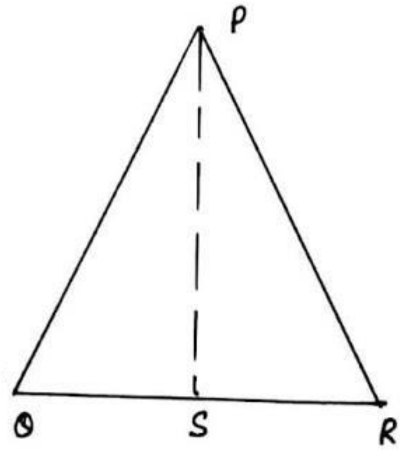
$\angle 2 + \angle 1 > \angle 4 + \angle 3$

$\boxed{\angle A > \angle C}$

Similarly by Joining BD, we can prove that

$\boxed{\angle B > \angle D}$

Q5 :→ In the figure $PR > PQ$. and PS bisects $\angle QPR$. Prove that $\angle PSR > \angle PSQ$



Sol :→ $PR > PQ$

$\angle PQR > \angle PRQ$ —(i)

[Angles opp to longer side is greater]

$\angle QPS > \angle RPS$ [∵ PS bisects $\angle QPR$] —(ii)

In ΔPQS, $\angle PQS + \angle QPS + \angle PSQ = 180°$

$\Rightarrow \angle PSQ = 180° - (\angle PQS + \angle QPS)$ —(iii)

Similarly in ΔPRS, $\angle PSR = 180 - (\angle PRS + \angle QPS)$ [from (ii)]

$\Rightarrow \angle PSR = 180^\circ - (\angle PRS + \angle QPS)$ — IV

From (i)

we know that $\angle PQS < \angle PSR$

So from (iii) and (iv), $\angle PSQ < \angle PSR$

$\Rightarrow \angle PSR > \angle PSQ$

Hence Proved

Q6 :→ Show that of all the segments drawn from a given point not on it the perpendicular line segment is the shortest.

Sol :→ We have A line $\overleftrightarrow{l}$ and O is a point not on $\overleftrightarrow{l}$

$OP \perp \overleftrightarrow{l}$

We have to prove that $OP < OQ$, $OP < OR$ and $OP < OS$

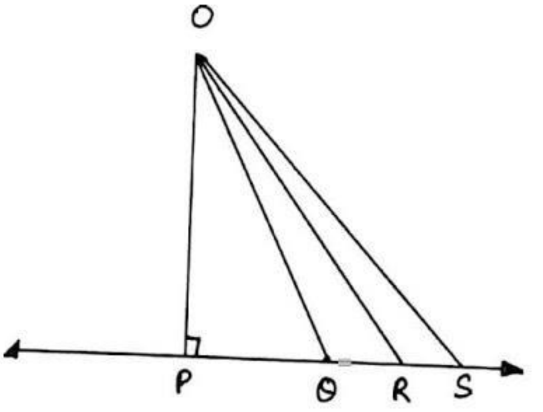
In ΔOPQ

$\angle P = 90^\circ$

$\therefore \angle Q$ is an acute angle

ie $\angle Q < 90^\circ$

$\therefore \angle Q < \angle P$



Hence $OP < OQ$

[side opposite to greater angle is longer]

Similarly, we can prove that OP is the shorter than OR, OS etc.

Hence Proved